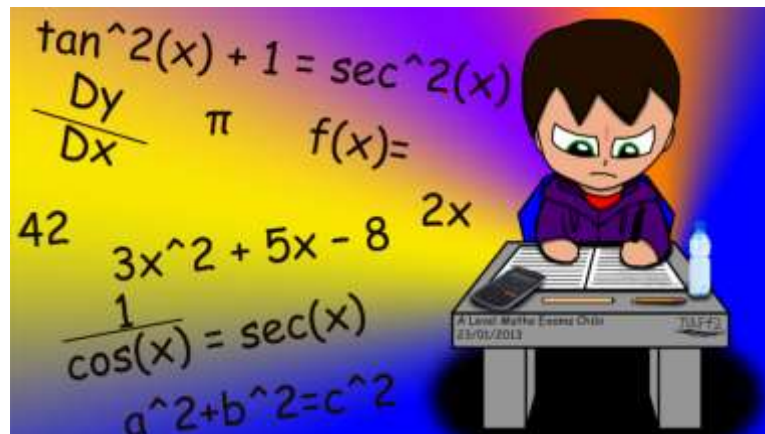


Huish Sixth

MATHS Department



Introduction to A-Level Maths

INDUCTION BOOKLET PART 1

PLEASE READ CAREFULLY

Thank you for choosing to study Mathematics in the sixth form at Huish Episcopi Academy. The Mathematics Department is committed to ensuring that you make good progress throughout your A level or AS course.

Due to Covid and finishing year 11 a little earlier than usual, you will not have been in a lesson environment for over 3 months by the time you start at Huish 6th. Therefore, it is vitaly important that you spend time working through the questions in this booklet over the course of the intervening weeks and months. The work you need to cover to prepare for A-level maths is in the form of two booklets this one and part 2 and the corresponding answer booklets for each. You should have met all the topics before at GCSE. Work through the introduction to each chapter, making sure that you understand the examples. Then tackle the exercise and attempt every question, you should write your solutions out on separate paper, (it might be worth getting a lined or squared book to keep everything in one place.) There is not room in the booklet to write solutions and your teacher next year will expect to see evidence that you have done this work and that you take pride in showing your method clearly. Once you have answered all the questions in a section you should mark your work by using the answer booklets. If you get a question wrong, you should re-try it and make sure you understand how to do it correctly.

Most of the topics have corresponding you-tube lessons that can be accessed from several websites, the best one to use for this is the mathedup website <http://www.mathedup.co.uk/transition-takeaway/>. These lessons should be your first port of call if you are unsure how to do a question. If you are still unable to do a question then you can use my maths, our login is **huish** and our password is **graph**. Another great website to visit is Maths Genie. In addition to this nearly all the topics can be found within a GCSE higher revision guide.

We will test you at the start of September to check how well you understand these topics, and that you have worked through the content of these booklet. It is important that you do some maths regularly at least something twice a week and not just all in one go before you start year 12, as you will need to be ready to tackle the content of the course, there will be no time to revise the content of the booklets. If you do not pass this test, then your suitability to study A-level maths will be questioned, especially if you fail to produce evidence that you have worked through both booklets. But as there is so much time to work through them I know this won't be an issue.

We hope that you will use this introduction to give you a good start to you're A-level work and that it will help you enjoy and benefit from the course more and we look forward to seeing you in September.

Mrs M Evans

Head of Maths

Contents of booklet 1

- 1 - Expanding brackets and simplifying expressions
- 2 - Surds and rationalising the denominator
- 3 - Rules of indices
- 4 - Factorising expressions
- 5 - Completing the square
- 6 - Solving quadratic equations by factorisation
- 7 - Sketching quadratic graphs
- 8 - Solving linear simultaneous equations using the elimination method
- 9 - Solving linear and quadratic simultaneous equations
- 10 - Solving simultaneous equations graphically
- 11 - Solving Linear Inequalities
- 12 - Solving Quadratic Inequalities

Expanding brackets and simplifying expressions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket
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Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	1 Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 2 Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$
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Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	1 Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 2 Simplify by collecting like terms: $2x + 3x = 5x$
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Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	1 Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 2 Simplify by collecting like terms: $3x - 10x = -7x$
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Practice

1 Expand.

a $3(2x - 1)$

c $-(3xy - 2y^2)$

b $-2(5pq + 4q^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$

c $9(3s + 1) - 5(6s - 10)$

b $8(5p - 2) - 3(4p + 9)$

d $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$

c $-2h(6h^2 + 11h - 5)$

b $4k(5k^2 - 12)$

d $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$

c $4p(2p - 1) - 3p(5p - 2)$

b $2x(x + 5) + 3x(x - 7)$

d $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a $13 - 2(m + 7)$

b $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x , for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$

$3x - 5$



$7x$

8 Expand and simplify.

a $(x + 4)(x + 5)$

c $(x + 7)(x - 2)$

e $(2x + 3)(x - 1)$

g $(5x - 3)(2x - 5)$

i $(3x + 4y)(5y + 6x)$

k $(2x - 7)^2$

b $(x + 7)(x + 3)$

d $(x + 5)(x - 5)$

f $(3x - 2)(2x + 1)$

h $(3x - 2)(7 + 4x)$

j $(x + 5)^2$

l $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$

b $\left(x + \frac{1}{x}\right)^2$

Watch out!

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '-'.

Surds and rationalising the denominator

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd \sqrt{b}
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$	<ol style="list-style-type: none">1 Choose two numbers that are factors of 50. One of the factors must be a square number2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$3 Use $\sqrt{25} = 5$
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Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\begin{aligned}\sqrt{147} - 2\sqrt{12} \\ &= \sqrt{49 \times 3} - 2\sqrt{4 \times 3} \\ \\ &= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3} \\ &= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} \\ &= 7\sqrt{3} - 4\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$	<ol style="list-style-type: none">1 Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$3 Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$4 Collect like terms
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Example 3 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$\begin{aligned} & (\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\ & = \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\ & = 7 - 2 \\ & = 5 \end{aligned}$	<ol style="list-style-type: none">1 Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$2 Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} = -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$
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Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\begin{aligned} \frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{1 \times \sqrt{3}}{\sqrt{9}} \\ &= \frac{\sqrt{3}}{3} \end{aligned}$	<ol style="list-style-type: none">1 Multiply the numerator and denominator by $\sqrt{3}$2 Use $\sqrt{9} = 3$
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Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\begin{aligned} \frac{\sqrt{2}}{\sqrt{12}} &= \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} \\ &= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12} \\ &= \frac{2\sqrt{2}\sqrt{3}}{12} \\ &= \frac{\sqrt{2}\sqrt{3}}{6} \end{aligned}$	<ol style="list-style-type: none">1 Multiply the numerator and denominator by $\sqrt{12}$2 Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$4 Use $\sqrt{4} = 2$5 Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$
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Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	<p>1 Multiply the numerator and denominator by $2-\sqrt{5}$</p> <p>2 Expand the brackets</p> <p>3 Simplify the fraction</p> <p>4 Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1</p>
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Practice

1 Simplify.

a $\sqrt{45}$

c $\sqrt{48}$

e $\sqrt{300}$

g $\sqrt{72}$

b $\sqrt{125}$

d $\sqrt{175}$

f $\sqrt{28}$

h $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72} + \sqrt{162}$

c $\sqrt{50} - \sqrt{8}$

e $2\sqrt{28} + \sqrt{28}$

b $\sqrt{45} - 2\sqrt{5}$

d $\sqrt{75} - \sqrt{48}$

f $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

c $(4 - \sqrt{5})(\sqrt{45} + 2)$

b $(3 + \sqrt{3})(5 - \sqrt{12})$

d $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$

b $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$

d $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$

f $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$

h $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$

b $\frac{2}{4+\sqrt{3}}$

c $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9} - \sqrt{8}}$

b $\frac{1}{\sqrt{x} - \sqrt{y}}$

Rules of indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none">1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<ol style="list-style-type: none">1 Use the rule $a^{-m} = \frac{1}{a^m}$2 Use $4^2 = 16$
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Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	<p>$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$</p>
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Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none">1 Use the rule $a^m \times a^n = a^{m+n}$2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	<p>Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged</p>
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<ol style="list-style-type: none">1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Practice

1 Evaluate.

a 14^0

b 3^0

c 5^0

d x^0

2 Evaluate.

a $49^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $125^{\frac{1}{3}}$

d $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$

b $8^{\frac{5}{3}}$

c $49^{\frac{3}{2}}$

d $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2}

b 4^{-3}

c 2^{-5}

d 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$

b $27^{-\frac{2}{3}}$

c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x .

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{\frac{3}{4}}$

9 Write the following in the form ax^n .

a $5\sqrt{x}$

b $\frac{2}{x^3}$

c $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$

e $\frac{4}{\sqrt[3]{x}}$

f 3

Extend

10 Write as sums of powers of x .

a $\frac{x^5 + 1}{x^2}$

b $x^2\left(x + \frac{1}{x}\right)$

c $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$

Factorising expressions

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac .
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets
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Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$
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Example 3 Factorise $x^2 + 3x - 10$

$b = 3, ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	<ol style="list-style-type: none">1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)2 Rewrite the b term ($3x$) using these two factors3 Factorise the first two terms and the last two terms4 $(x + 5)$ is a factor of both terms
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Example 4 Factorise $6x^2 - 11x - 10$

$b = -11, ac = -60$ <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms
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Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator: $b = -4, ac = -21$</p> <p>So</p> $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$ <p>For the denominator: $b = 9, ac = 18$</p> <p>So</p> $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ <p>So</p> $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> 1 Factorise the numerator and the denominator 2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) 3 Rewrite the b term ($-4x$) using these two factors 4 Factorise the first two terms and the last two terms 5 $(x - 7)$ is a factor of both terms 6 Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) 7 Rewrite the b term ($9x$) using these two factors 8 Factorise the first two terms and the last two terms 9 $(x + 3)$ is a factor of both terms 10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
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Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$
c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b $21a^3b^5 + 35a^5b^2$

Hint

Take the highest common factor outside the bracket.

2 Factorise

a $x^2 + 7x + 12$
c $x^2 - 11x + 30$
e $x^2 - 7x - 18$
g $x^2 - 3x - 40$

b $x^2 + 5x - 14$
d $x^2 - 5x - 24$
f $x^2 + x - 20$
h $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$
c $18a^2 - 200b^2c^2$

b $4x^2 - 81y^2$

4 Factorise

a $2x^2 + x - 3$
c $2x^2 + 7x + 3$
e $10x^2 + 21x + 9$

b $6x^2 + 17x + 5$
d $9x^2 - 15x + 4$
f $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\frac{2x^2 + 4x}{x^2 - x}$

b $\frac{x^2 + 3x}{x^2 + 2x - 3}$

c $\frac{x^2 - 2x - 8}{x^2 - 4x}$

d $\frac{x^2 - 5x}{x^2 - 25}$

e $\frac{x^2 - x - 12}{x^2 - 4x}$

f $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	<p>1 Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$</p> <p>2 Simplify</p>
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Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	<p>1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$</p> <p>2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$</p> <p>3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2</p> <p>4 Simplify</p>
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Practice

- 1 Write the following quadratic expressions in the form $(x + p)^2 + q$
- | | |
|-------------------------|--------------------------|
| a $x^2 + 4x + 3$ | b $x^2 - 10x - 3$ |
| c $x^2 - 8x$ | d $x^2 + 6x$ |
| e $x^2 - 2x + 7$ | f $x^2 + 3x - 2$ |
- 2 Write the following quadratic expressions in the form $p(x + q)^2 + r$
- | | |
|---------------------------|---------------------------|
| a $2x^2 - 8x - 16$ | b $4x^2 - 8x - 16$ |
| c $3x^2 + 12x - 9$ | d $2x^2 + 6x - 8$ |
- 3 Complete the square.
- | | |
|--------------------------|--------------------------|
| a $2x^2 + 3x + 6$ | b $3x^2 - 2x$ |
| c $5x^2 + 3x$ | d $3x^2 + 5x + 3$ |

Extend

- 4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Solving quadratic equations by factorisation

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	<ol style="list-style-type: none">1 Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.2 Factorise the quadratic equation. $5x$ is a common factor.3 When two values multiply to make zero, at least one of the values must be zero.4 Solve these two equations.
---	--

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7, ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	<ol style="list-style-type: none">1 Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)2 Rewrite the b term ($7x$) using these two factors.3 Factorise the first two terms and the last two terms.4 $(x + 4)$ is a factor of both terms.5 When two values multiply to make zero, at least one of the values must be zero.6 Solve these two equations.
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Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	<ol style="list-style-type: none">1 Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.2 When two values multiply to make zero, at least one of the values must be zero.3 Solve these two equations.
---	---

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	<ol style="list-style-type: none">1 Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)2 Rewrite the b term ($-5x$) using these two factors.3 Factorise the first two terms and the last two terms.4 $(x - 4)$ is a factor of both terms.5 When two values multiply to make zero, at least one of the values must be zero.6 Solve these two equations.
--	--

Practice

1 Solve

a $6x^2 + 4x = 0$

c $x^2 + 7x + 10 = 0$

e $x^2 - 3x - 4 = 0$

g $x^2 - 10x + 24 = 0$

i $x^2 + 3x - 28 = 0$

k $2x^2 - 7x - 4 = 0$

b $28x^2 - 21x = 0$

d $x^2 - 5x + 6 = 0$

f $x^2 + 3x - 10 = 0$

h $x^2 - 36 = 0$

j $x^2 - 6x + 9 = 0$

l $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$

c $x^2 + 5x = 24$

e $x(x + 2) = 2x + 25$

g $x(3x + 1) = x^2 + 15$

b $x^2 - 3 = 2x$

d $x^2 - 42 = x$

f $x^2 - 30 = 3x - 2$

h $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$ $(x + 3)^2 - 9 + 4 = 0$ $(x + 3)^2 - 5 = 0$ $(x + 3)^2 = 5$ $x + 3 = \pm\sqrt{5}$ $x = \pm\sqrt{5} - 3$ <p>So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$</p>	<ol style="list-style-type: none"> Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ Simplify. Rearrange the equation to work out x. First, add 5 to both sides. Square root both sides. Remember that the square root of a value gives two answers. Subtract 3 from both sides to solve the equation. Write down both solutions.
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Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$ $2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$ $2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$ $2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$ $2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	<ol style="list-style-type: none"> Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$ Expand the square brackets. Simplify. <p style="text-align: right;"><i>(continued on next page)</i></p>
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$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$ $x - \frac{7}{4} = \pm \frac{\sqrt{17}}{4}$ $x = \pm \frac{\sqrt{17}}{4} + \frac{7}{4}$ <p>So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$</p>	<p>5 Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.</p> <p>6 Divide both sides by 2.</p> <p>7 Square root both sides. Remember that the square root of a value gives two answers.</p> <p>8 Add $\frac{7}{4}$ to both sides.</p> <p>9 Write down both the solutions.</p>
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Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$

b $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$

d $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$

f $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a , b and c .

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ $x = \frac{-6 \pm \sqrt{20}}{2}$ $x = \frac{-6 \pm 2\sqrt{5}}{2}$ $x = -3 \pm \sqrt{5}$ So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	<ol style="list-style-type: none">Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.Substitute $a = 1, b = 6, c = 4$ into the formula.Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$Simplify by dividing numerator and denominator by 2.Write down both the solutions.
--	---

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{7 \pm \sqrt{73}}{6}$ <p>So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$</p>	<ol style="list-style-type: none">1 Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.2 Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.4 Write down both the solutions.
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Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$

b $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

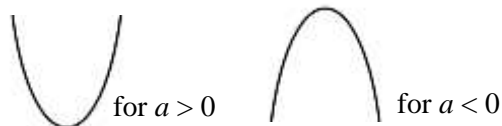
Sketching quadratic graphs

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.



Examples

Example 1 Sketch the graph of $y = x^2$.

	<p>The graph of $y = x^2$ is a parabola.</p> <p>When $x = 0$, $y = 0$.</p> <p>$a = 1$ which is greater than zero, so the graph has the shape:</p>
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Example 2 Sketch the graph of $y = x^2 - x - 6$.

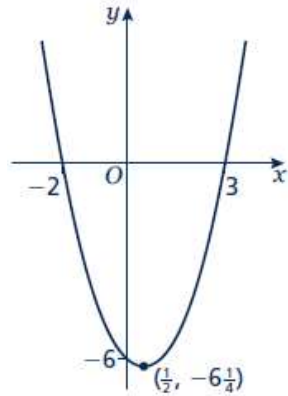
<p>When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$ When $y = 0$, $x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$ $x = -2$ or $x = 3$ So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$</p> $x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$	<ol style="list-style-type: none"> Find where the graph intersects the y-axis by substituting $x = 0$. Find where the graph intersects the x-axis by substituting $y = 0$. Solve the equation by factorising. Solve $(x + 2) = 0$ and $(x - 3) = 0$. $a = 1$ which is greater than zero, so the graph has the shape: To find the turning point, complete the square. <p style="text-align: right;"><i>(continued on next page)</i></p>
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$$= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$$

When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and

$y = -\frac{25}{4}$, so the turning point is at the

point $\left(\frac{1}{2}, -\frac{25}{4}\right)$



7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

- Sketch the graph of $y = -x^2$.
- Sketch each graph, labelling where the curve crosses the axes.
 - $y = (x + 2)(x - 1)$
 - $y = x(x - 3)$
 - $y = (x + 1)(x + 5)$
- Sketch each graph, labelling where the curve crosses the axes.
 - $y = x^2 - x - 6$
 - $y = x^2 - 5x + 4$
 - $y = x^2 - 4$
 - $y = x^2 + 4x$
 - $y = 9 - x^2$
 - $y = x^2 + 2x - 3$
- Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

- Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.
 - $y = x^2 - 5x + 6$
 - $y = -x^2 + 7x - 12$
 - $y = -x^2 + 4x$
- Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \\ \text{So } x = 2 \end{array}$	1 Subtract the second equation from the first equation to eliminate the y term.
$\begin{array}{r} \text{Using } x + y = 1 \\ \quad 2 + y = 1 \\ \text{So } y = -1 \end{array}$	2 To find the value of y , substitute $x = 2$ into one of the original equations.
<p>Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES</p>	3 Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \\ \text{So } x = 3 \end{array}$	1 Add the two equations together to eliminate the y term.
$\begin{array}{r} \text{Using } x + 2y = 13 \\ \quad 3 + 2y = 13 \\ \text{So } y = 5 \end{array}$	2 To find the value of y , substitute $x = 3$ into one of the original equations.
<p>Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES</p>	3 Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow \frac{15x + 12y = 36}{7x = 28} \end{array}$	<p>1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.</p> <p>2 To find the value of y, substitute $x = 4$ into one of the original equations.</p> <p>3 Substitute the values of x and y into both equations to check your answers.</p>
<p>So $x = 4$</p> <p>Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$</p> <p>Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES</p>	

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
 $x + y = 5$

2 $3x + y = 7$
 $3x + 2y = 5$

3 $4x + y = 3$
 $3x - y = 11$

4 $3x + 4y = 7$
 $x - 4y = 5$

5 $2x + y = 11$
 $x - 3y = 9$

6 $2x + 3y = 11$
 $3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ $\text{So } x = 1$ $\text{Using } y = 2x + 1$ $y = 2 \times 1 + 1$ $\text{So } y = 3$ Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	<ol style="list-style-type: none">1 Substitute $2x + 1$ for y into the second equation.2 Expand the brackets and simplify.3 Work out the value of x.4 To find the value of y, substitute $x = 1$ into one of the original equations.5 Substitute the values of x and y into both equations to check your answers.
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Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ $\text{So } x = 4\frac{1}{2}$ $\text{Using } y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ $\text{So } y = -7$ Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	<ol style="list-style-type: none">1 Rearrange the first equation.2 Substitute $2x - 16$ for y into the second equation.3 Expand the brackets and simplify.4 Work out the value of x.5 To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.6 Substitute the values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

7 $y = x - 4$
 $2x + 5y = 43$

8 $y = 2x - 3$
 $5x - 3y = 11$

9 $2y = 4x + 5$
 $9x + 5y = 22$

10 $2x = y - 2$
 $8x - 5y = -11$

11 $3x + 4y = 8$
 $2x - y = -13$

12 $3y = 4x - 7$
 $2y = 3x - 4$

13 $3x = y - 1$
 $2y - 2x = 3$

14 $3x + 2y + 1 = 0$
 $4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Solving linear and quadratic simultaneous equations

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$ $2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ So $x = 2$ or $x = -3$ Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$ So the solutions are $x = 2, y = 3$ and $x = -3, y = -2$ Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	<ol style="list-style-type: none">1 Substitute $x + 1$ for y into the second equation.2 Expand the brackets and simplify.3 Factorise the quadratic equation.4 Work out the values of x.5 To find the value of y, substitute both values of x into one of the original equations.6 Substitute both pairs of values of x and y into both equations to check your answers.
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Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$ $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ $2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$ $(y+8)(y-3) = 0$ <p>So $y = -8$ or $y = 3$</p> <p>Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$</p> <p>So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$</p> <p>Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES</p>	<ol style="list-style-type: none"> 1 Rearrange the first equation. 2 Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y. 3 Expand the brackets and simplify. 4 Factorise the quadratic equation. 5 Work out the values of y. 6 To find the value of x, substitute both values of y into one of the original equations. 7 Substitute both pairs of values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
 $x^2 + y^2 = 10$

2 $y = 6 - x$
 $x^2 + y^2 = 20$

3 $y = x - 3$
 $x^2 + y^2 = 5$

4 $y = 9 - 2x$
 $x^2 + y^2 = 17$

5 $y = 3x - 5$
 $y = x^2 - 2x + 1$

6 $y = x - 5$
 $y = x^2 - 5x - 12$

7 $y = x + 5$
 $x^2 + y^2 = 25$

8 $y = 2x - 1$
 $x^2 + xy = 24$

9 $y = 2x$
 $y^2 - xy = 8$

10 $2x + y = 11$
 $xy = 15$

Extend

11 $x - y = 1$
 $x^2 + y^2 = 3$

12 $y - x = 2$
 $x^2 + xy = 3$

Solving simultaneous equations graphically

A LEVEL LINKS

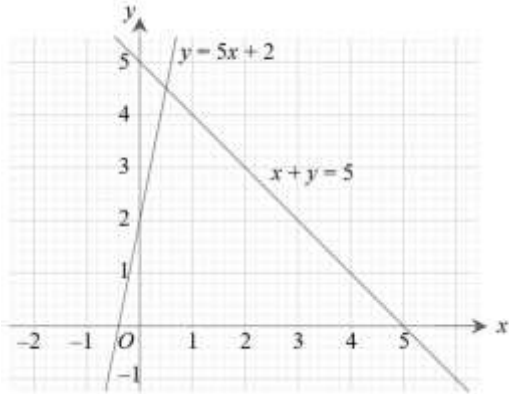
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

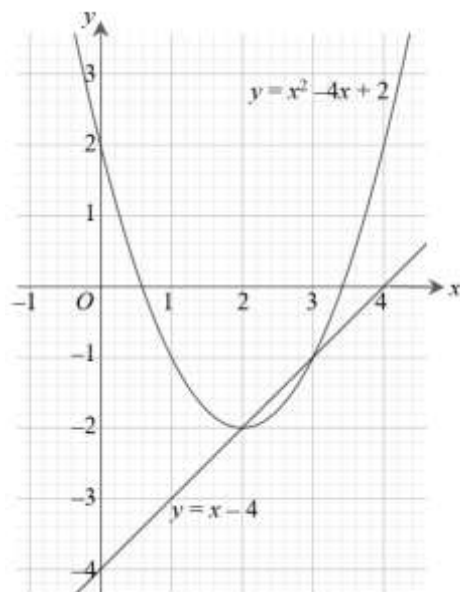
Examples

Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

<p>$y = 5 - x$</p> <p>$y = 5 - x$ has gradient -1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2.</p>  <p>Lines intersect at $x = 0.5, y = 4.5$</p> <p>Check: First equation $y = 5x + 2$: $4.5 = 5 \times 0.5 + 2$ YES Second equation $x + y = 5$: $0.5 + 4.5 = 5$ YES</p>	<ol style="list-style-type: none">Rearrange the equation $x + y = 5$ to make y the subject.Plot both graphs on the same grid using the gradients and y-intercepts.The solutions of the simultaneous equations are the point of intersection.Check your solutions by substituting the values into both equations.
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Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	-1	-2	-1	2



The line and curve intersect at
 $x = 3, y = -1$ and $x = 2, y = -2$

Check:

First equation $y = x - 4$:

$$-1 = 3 - 4 \quad \text{YES}$$

$$-2 = 2 - 4 \quad \text{YES}$$

Second equation $y = x^2 - 4x + 2$:

$$-1 = 3^2 - 4 \times 3 + 2 \quad \text{YES}$$

$$-2 = 2^2 - 4 \times 2 + 2 \quad \text{YES}$$

- 1 Construct a table of values and calculate the points for the quadratic equation.
- 2 Plot the graph.
- 3 Plot the linear graph on the same grid using the gradient and y-intercept.
 $y = x - 4$ has gradient 1 and y-intercept -4 .
- 4 The solutions of the simultaneous equations are the points of intersection.
- 5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

a $y = x - 1$ and $y = x^2 - 4x + 3$

b $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 a Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

i graphically

ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Linear inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$\begin{aligned} -8 &\leq 4x < 16 \\ -2 &\leq x < 4 \end{aligned}$	Divide all three terms by 4.
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Example 2 Solve $4 \leq 5x < 10$

$\begin{aligned} 4 &\leq 5x < 10 \\ \frac{4}{5} &\leq x < 2 \end{aligned}$	Divide all three terms by 5.
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Example 3 Solve $2x - 5 < 7$

$\begin{aligned} 2x - 5 &< 7 \\ 2x &< 12 \\ x &< 6 \end{aligned}$	<ol style="list-style-type: none">1 Add 5 to both sides.2 Divide both sides by 2.
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Example 4 Solve $2 - 5x \geq -8$

$\begin{aligned} 2 - 5x &\geq -8 \\ -5x &\geq -10 \\ x &\leq 2 \end{aligned}$	<ol style="list-style-type: none">1 Subtract 2 from both sides.2 Divide both sides by -5. Remember to reverse the inequality when dividing by a negative number.
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Example 5 Solve $4(x - 2) > 3(9 - x)$

$\begin{aligned} 4(x - 2) &> 3(9 - x) \\ 4x - 8 &> 27 - 3x \\ 7x - 8 &> 27 \\ 7x &> 35 \\ x &> 5 \end{aligned}$	<ol style="list-style-type: none">1 Expand the brackets.2 Add $3x$ to both sides.3 Add 8 to both sides.4 Divide both sides by 7.
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Practice

1 Solve these inequalities.

a $4x > 16$

b $5x - 7 \leq 3$

c $1 \geq 3x + 4$

d $5 - 2x < 12$

e $\frac{x}{2} \geq 5$

f $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$

b $10 \geq 2x + 3$

c $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$

b $3 \leq 7x + 10 < 45$

c $6 - 2x \geq 4$

d $4x + 17 < 2 - x$

e $4 - 5x < -3x$

f $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$

b $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$

b $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Quadratic inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

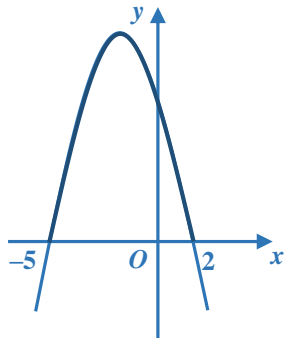
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3 \text{ or } x = -2$ <p>$x < -3 \text{ or } x > -2$</p>	<ol style="list-style-type: none">1 Solve the quadratic equation by factorising.2 Sketch the graph of $y = (x + 3)(x + 2)$3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$
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Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0 \text{ or } x = 5$ <p>$0 \leq x \leq 5$</p>	<ol style="list-style-type: none">1 Solve the quadratic equation by factorising.2 Sketch the graph of $y = x(x - 5)$3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$
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Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2 \text{ or } x = -5$  $-5 \leq x \leq 2$	<ol style="list-style-type: none"> 1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$ 3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ 3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$
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Practice

- 1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

- 6 $x^2 + x \leq 6$
- 7 $x(2x - 9) < -10$
- 8 $6x^2 \geq 15 + x$